AP1 Oscillations

1. Which of the following statements about a spring-block oscillator in simple harmonic motion about its equilibrium point is false?

(A) The displacement is directly related to the acceleration.
(B) The acceleration and velocity vectors always point in the same direction.
(C) The acceleration vector is always toward the equilibrium point.
(D) The acceleration and displacement vectors always point in opposite directions.

Answer: (B) The acceleration and velocity vectors always point in the same direction.

A, C, and D are all true, but the acceleration and velocity vectors sometimes point in the same direction, and sometimes point in opposite directions.

EK: 3.B.3 Restoring forces can result in oscillatory motion. When a linear restoring force is exerted on an object displaced from an equilibrium position, the object will undergo a special type of motion called simple harmonic motion.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 3.3 The student can evaluate scientific questions. 6.1 The student can justify claims with evidence. 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models. 6.5 The student can evaluate alternative scientific explanations.

LO: 3.B.3.1 The student is able to predict which properties determine the motion of a simple harmonic oscillator and what the dependence of the motion is on those properties. 3.B.3.3 The student can analyze data to identify qualitative or quantitative relationships between given values and variables (i.e., force, displacement, acceleration, velocity, period of motion, frequency, spring constant, string length, mass) associated with objects in oscillatory motion to use that data to determine the value of an unknown.

AP1 Oscillations

2. Which of the following are most likely to result in simple harmonic motion? Select two answers.

(A) A hole is drilled through one end of a meter stick, which is hung vertically from a frictionless axle. The bottom of the meter stick is displaced 12 degrees and released.
(B) A hole is drilled through one end of a meter stick, which is hung vertically from a rough axle. The bottom of the meter stick is displaced 12 degrees and released. Every time the meter stick swings back and forth the axle squeaks.
(C) A block is hung vertically from a linear spring. The opposite end of the spring is attached to a stationary point. The entire apparatus is placed in deep space. The block is displaced 4 cm from equilibrium and released.
(D) A block is placed on a frictionless surface and attached to a non-linear spring. The opposite end of the spring is attached to a wall. The block is displaced 2 cm from equilibrium and released.

Answers: (A) and (C). Simple harmonic motion requires a linear restoring force. The real pendulum described in answer (A) can be readily approximated using simple harmonic motion using the small angle approximation. Answer (B) can be eliminated due to the loss of energy from the rough axle, which will result in damping. Answer (C) describes true simple harmonic motion in a frictionless environment with the spring-block oscillator. Answer (D) can be eliminated due to the non-linear spring.

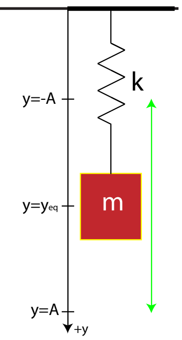
EK: 3.B.3 Restoring forces can result in oscillatory motion. When a linear restoring force is exerted on an object displaced from an equilibrium position, the object will undergo a special type of motion called simple harmonic motion.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models.

LO: 3.B.3.1 The student is able to predict which properties determine the motion of a simple harmonic oscillator and what the dependence of the motion is on those properties. 3.B.3.4 The student is able to construct a qualitative and/or a quantitative explanation of oscillatory behavior given evidence of a restoring force.

AP1 Oscillations

3. A spring of spring constant k is hung vertically from a fixed surface, and a block of mass M is attached to the bottom of the spring. The mass is released and the system is allowed to come to equilibrium as shown in the diagram at right.



(a) Derive an expression for the equilibrium position of the mass.

(b) The spring is now pulled downward and displaced an amount A. Derive an expression for the potential energy stored in the spring.

(c) At time t=0, the spring is released. Derive an expression for the period of the spring-block oscillator.

(d) Describe an experimental procedure you could use to verify your derivation of the period. Include all equipment required.

(e) How would you analyze the data to determine whether the experimental data verifies your derivation? What evidence from the analysis would be used to make the determination?

Answer:

(a) $y_{eq} = \dfrac{Mg}{k}$

(b) $U_s = \dfrac{1}{2}kA^2$

(c) $T_s = 2\pi\sqrt{\dfrac{M}{k}}$

(d) One method could involve utilizing a stopwatch to measure the time it takes for 10 oscillations, and dividing that by 10 to obtain the experimental period for the given mass. A more detailed analysis could include repeating this for a variety of masses. Equipment required would include a stopwatch and hanging masses. A variety of alternate acceptable answers exist, which could include, but are not limited to, electronic measuring devices such as photogates.

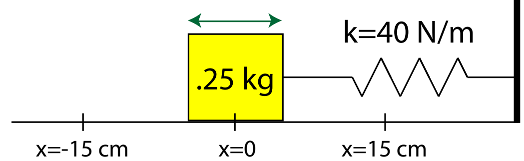
(e) One method would involve plotting the period vs. the square root of the mass. This plot should be linear, with the slope equal to 2π divided by $\sqrt{k}$. Including an uncertainty analysis of the data, if the calculated value for the spring constant from the slope of the graph matches the spring constant of the spring (within its uncertainty), it would be reasonable to conclude that the derivation is correct.

EK: 3.B.3 Restoring forces can result in oscillatory motion. When a linear restoring force is exerted on an object displaced from an equilibrium position, the object will undergo a special type of motion called simple harmonic motion. 4.C.1 The energy of a system includes its kinetic energy, potential energy, and microscopic internal energy. Examples should include gravitational potential energy, elastic potential energy, and kinetic energy.

SP: 3.3 The student can evaluate scientific questions. 4.1 The student can justify the selection of the kind of data needed to answer a particular scientific question. 4.2 The student can design a plan for collecting data to answer a particular scientific question. 5.1 The student can analyze data to identify patterns or relationships.

LO: 3.B.3.1 The student is able to predict which properties determine the motion of a simple harmonic oscillator and what the dependence of the motion is on those properties. 3.B.3.2 The student is able to design a plan and collect data in order to ascertain the characteristics of the motion of a system undergoing oscillatory motion caused by a restoring force. 3.B.3.3 The student can analyze data to identify qualitative or quantitative relationships between given values and variables (i.e., force, displacement, acceleration, velocity, period of motion, frequency, spring constant, string length, mass) associated with objects in oscillatory motion to use that data to determine the value of an unknown. 4.C.1.1 The student is able to calculate the total energy of a system and justify the mathematical routines used in the calculation of component types of energy within the system whose sum is the total energy.

4. A spring of spring constant 40 N/m is attached to a fixed surface, and a block of mass 0.25 kg is attached to the end of the spring, sitting on a frictionless surface. The block is now displaced 15 centimeters and released at time t=0.



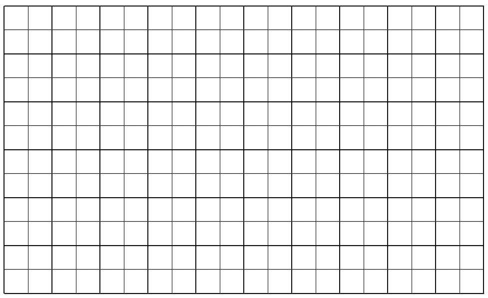
(a) Draw a free body diagram for the block at t=0.

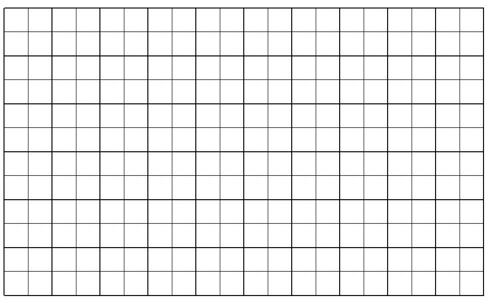
(b) Determine the period of the spring-block oscillator.

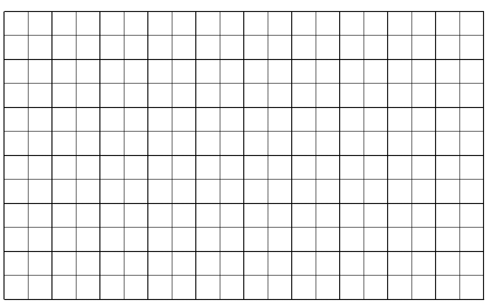
(c) Determine the speed of the block at position x=0.

(d) Write an expression for the displacement of the block as a function of time.

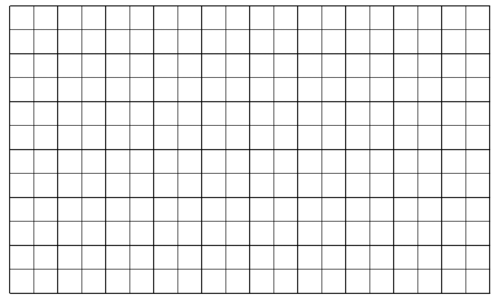
(e) On the graphs below, plot the displacement, speed, and acceleration of the mass as a function of time. Explicitly label axes with units as well as any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.





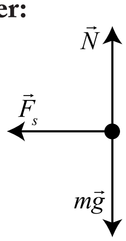


(f) On the axes below, plot the kinetic energy of the mass and the elastic potential energy of the spring as functions of time. Label your plots K and U_s. Explicitly label axes with units as well as any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



Answer:

(a)

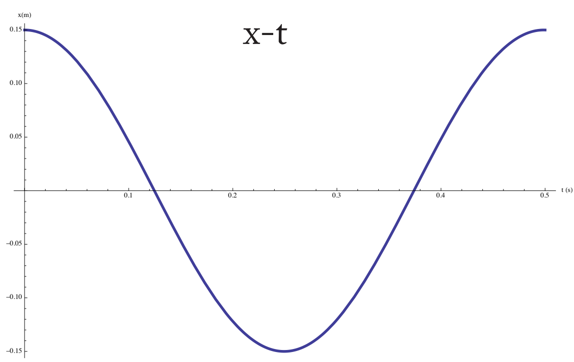


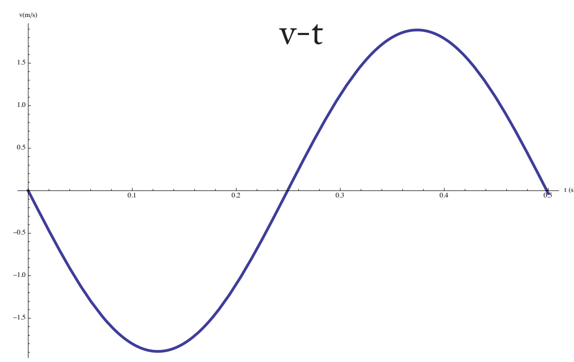
(b) $T_s = 2\pi\sqrt{\frac{M}{k}} = 0.497s$

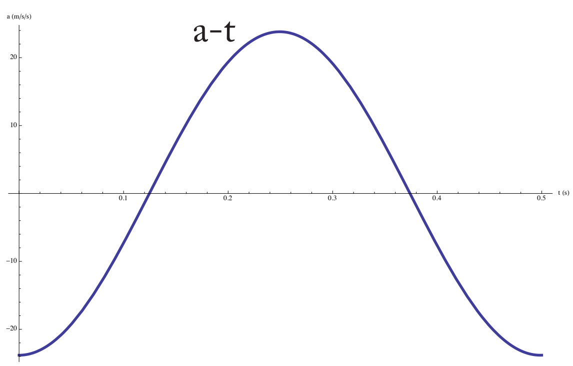
(c) $\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{kx^2}{m}} = \sqrt{\frac{(40)(.15)^2}{.25}} = 1.90\,{}^{m}\!/_{s}$

(d) $x(t) = A\cos(\omega t) \rightarrow x(t) = 0.15\cos(12.6t)$

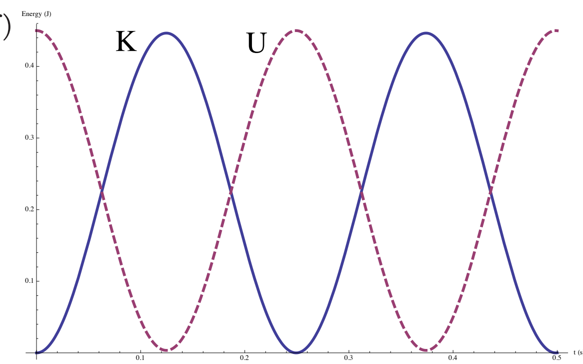
(e)







(f)



EK: 3.B.3 Restoring forces can result in oscillatory motion. When a linear restoring force is exerted on an object displaced from an equilibrium position, the object will undergo a special type of motion called simple harmonic motion. 4.C.1 The energy of a system includes its kinetic energy, potential energy, and microscopic internal energy. Examples should include gravitational potential energy, elastic potential energy, and kinetic energy.

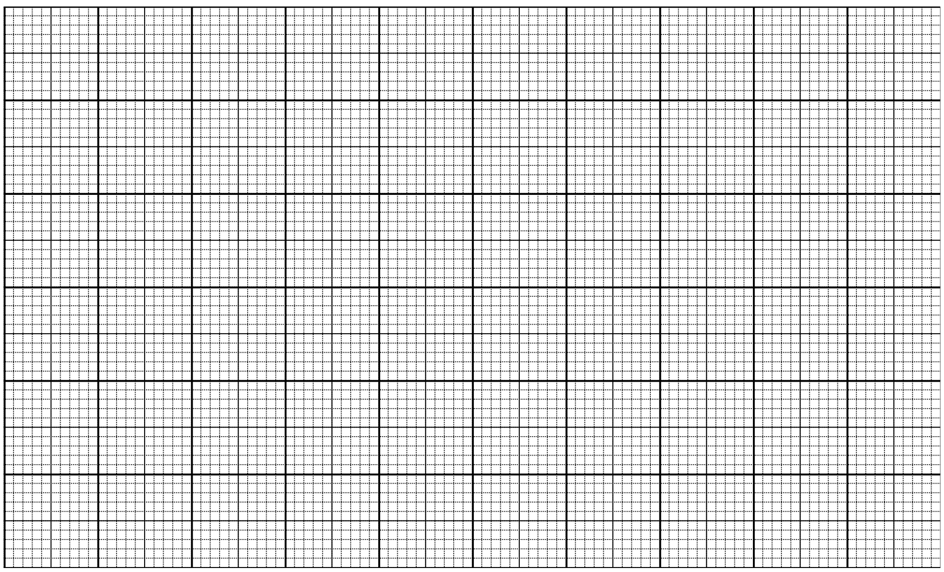
SP: 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 3.3 The student can evaluate scientific questions. 5.1 The student can analyze data to identify patterns or relationships. 5.3 The student can evaluate the evidence provided by data sets in relation to a particular scientific question.

LO: 3.B.3.1 The student is able to predict which properties determine the motion of a simple harmonic oscillator and what the dependence of the motion is on those properties. 3.B.3.2 The student is able to design a plan and collect data in order to ascertain the characteristics of the motion of a system undergoing oscillatory motion caused by a restoring force. 3.B.3.3 The student can analyze data to identify qualitative or quantitative relationships between given values and variables (i.e., force, displacement, acceleration, velocity, period of motion, frequency, spring constant, string length, mass) associated with objects in oscillatory motion to use that data to determine the value of an unknown. 4.C.1.2 The student is able to predict changes in the total energy of a system due to changes in position and speed of objects or frictional interactions within the system.

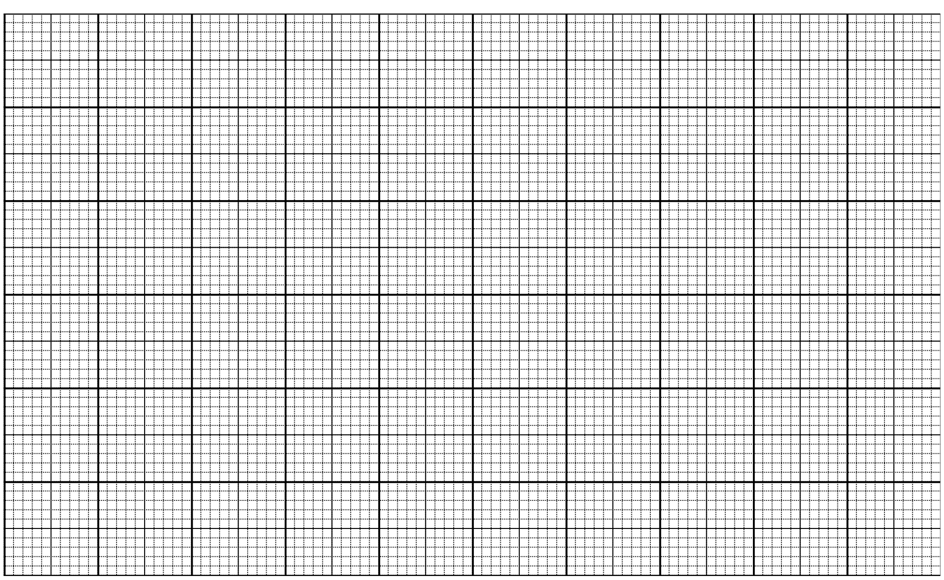
5. Students are to conduct an experiment to investigate the relationship between the length of a pendulum and its period. Their procedure involves hanging a 0.5 kg mass on a string of varying length, L, setting it into oscillation, and measuring the period. The students conduct the experiment and obtain the following data.

Trial	1	2	3	4	5	6
Length (m)	0.25	0.50	0.75	1.0	1.5	2.0
Period (s)	1.0	1.4	1.7	2.0	2.5	2.8

(a) On the grid below, plot the period of the pendulum T as a function of the length of the string, L, and draw a best-fit curve. Label the axes as appropriate.



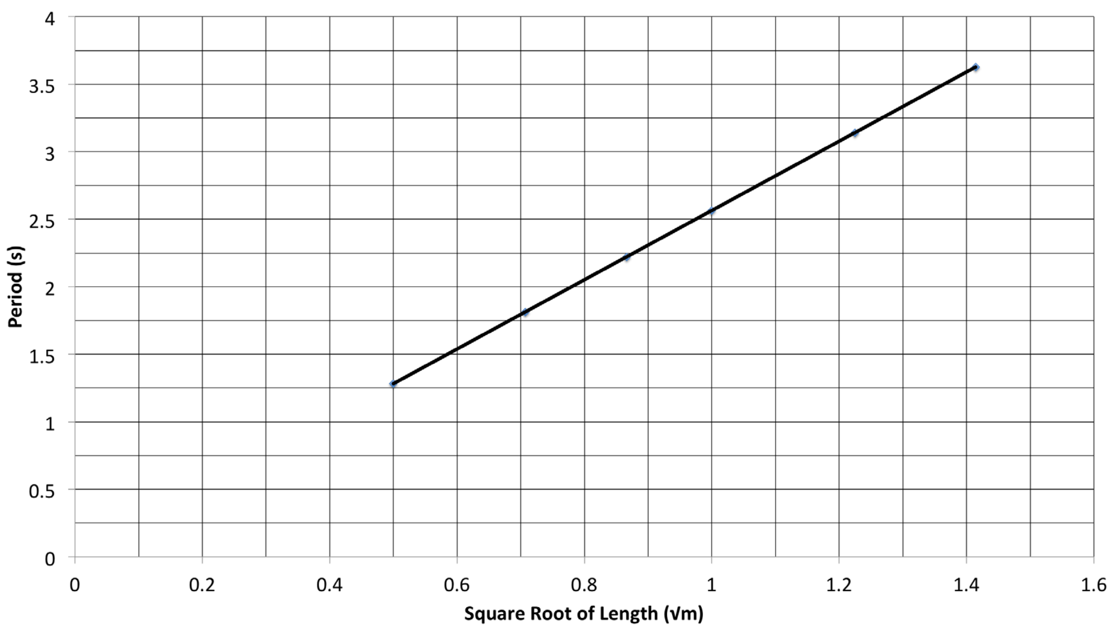
(b) Use the grid below to plot a linear graph as a function of L. Use the empty boxes in the data table to record any calculated values you are graphing. Label the axes as appropriate.



Students are then given strings of various lengths, a hanging mass, a meter stick, a stopwatch, appropriate survival equipment, and are transported to the surface of Planet X. There, they are asked to determine the acceleration due to gravity on the surface of Planet X using just this equipment.

(c) Describe an experimental procedure that the student could use to collect the necessary data as accurately as possible.

In order to determine the acceleration due to gravity, the students then create a plot of T vs. the square root of L, as shown below.



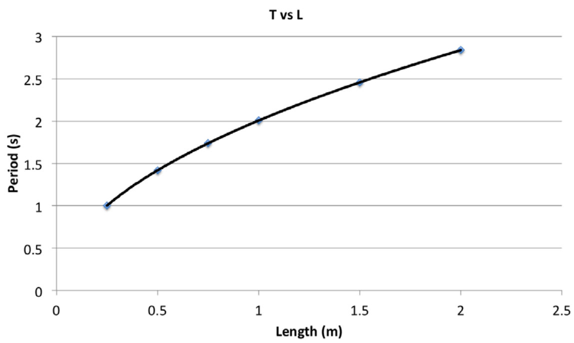
(d) Using the graph, calculate the acceleration due to gravity on Planet X.

(e) Following further analysis and experiments, a comparison of the students' experimental value for the acceleration due to gravity on Planet X is greater than the actual acceleration due to gravity on Planet X. Offer a reasonable explanation for this difference.

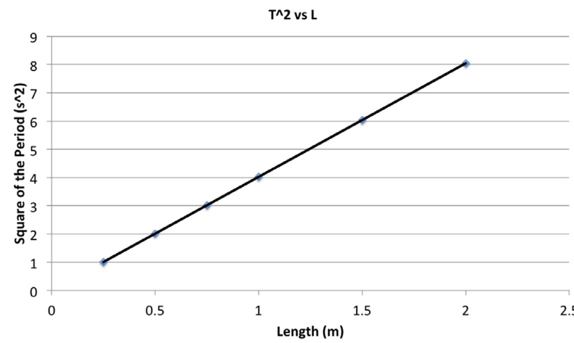
AP1 Oscillations

Answer:

(a)



(b)



(c) Hang the mass from the string and attach the top of the string to a fixed point. Pull the mass back a small angular displacement (~ 10 degrees) and release. Use a stopwatch to time 20 oscillations, and divide the total time by 20 to obtain the period.

(d) $g = \frac{4\pi^2}{slope^2} = \frac{4\pi^2}{(2.5651)^2} = 6 \, m/s^2$

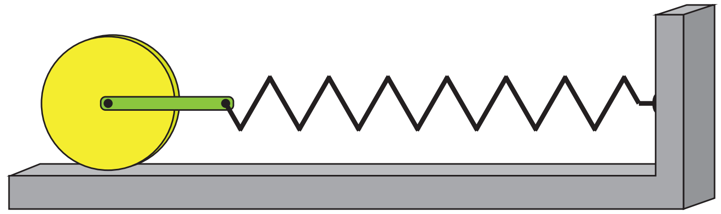
(e) Our analysis assumes the string is massless. A real string has mass, which would lead to a smaller measured period of oscillation, and therefore a larger measured acceleration due to gravity compared with the actual acceleration due to gravity on the planet.

EK: 3.B.3 Restoring forces can result in oscillatory motion. When a linear restoring force is exerted on an object displaced from an equilibrium position, the object will undergo a special type of motion called simple harmonic motion.

SP: 3.3 The student can evaluate scientific questions. 4.2 The student can design a plan for collecting data to answer a particular scientific question. 5.1 The student can analyze data to identify patterns or relationships. 5.3 The student can evaluate the evidence provided by data sets in relation to a particular scientific question. 6.5 The student can evaluate alternative scientific explanations.

LO: 3.B.3.1 The student is able to predict which properties determine the motion of a simple harmonic oscillator and what the dependence of the motion is on those properties. 3.B.3.2 The student is able to design a plan and collect data in order to ascertain the characteristics of the motion of a system undergoing oscillatory motion caused by a restoring force. 3.B.3.3 The student can analyze data to identify qualitative or quantitative relationships between given values and variables (i.e., force, displacement, acceleration, velocity, period of motion, frequency, spring constant, string length, mass) associated with objects in oscillatory motion to use that data to determine the value of an unknown. 3.B.3.4 The student is able to construct a qualitative and/or a quantitative explanation of oscillatory behavior given evidence of a restoring force.

6. A disk with a mass M, a radius R, and a rotational inertia of $I = \frac{1}{2} MR^2$ is attached to a horizontal spring which has a spring constant of k as shown in the diagram. When the spring is stretched by a distance x and then released from rest, the disk rolls without slipping while the spring is attached to the frictionless axle within the center of the disk.



(a) Calculate the maximum translational velocity of the disk in terms of M,R, x, k.

(b) What would happen to the period of this motion if the spring constant of the spring increased? Justify your answer.

(c) What would happen to the period of this motion if the surface was now frictionless and the disk was not allowed to roll? Justify your answer.

Answer:

(a) Use conservation of energy to find the maximum velocity: $\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \xrightarrow[\omega=\frac{v}{R}]{I=\frac{1}{2}MR^2} v_{\max} = x\sqrt{\frac{2k}{3M}}$

(b) If the spring constant k increased, the period of the motion would decrease. When you increase the k value of a spring the period of the spring decreases. This is due to the velocity of the object attached to the spring increasing. You can also look at the answer in part (a) and see that the velocity increases with a larger k which means the time to travel one full oscillation will decrease.

(c) There is a decrease in the period if the system is now on a frictionless surface. The disk will no longer roll and no energy is being put into rolling the disk. The total energy of the system will stay the same but more energy is now available for the translational kinetic energy (assuming we stretch the spring the same distance x). This leads to a higher velocity, which leads to a decrease in the period.

EK: 3.B.3 Restoring forces can result in oscillatory motion. When a linear restoring force is exerted on an object displaced from an equilibrium position, the object will undergo a special type of motion called simple harmonic motion. 5.A.2 For all systems under all circumstances, energy, charge, linear momentum, and angular momentum are conserved. For an isolated or a closed system, conserved quantities are constant. An open system is one that exchanges any conserved quantity with its surroundings.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 3.3 The student can evaluate scientific questions. 6.1 The student can justify claims with evidence. 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models.

LO: 3.B.3.1 The student is able to predict which properties determine the motion of a simple harmonic oscillator and what the dependence of the motion is on those properties. 3.B.3.4 The student is able to construct a qualitative and/or a quantitative explanation of oscillatory behavior given evidence of a restoring force. 5.A.2.1 The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations.

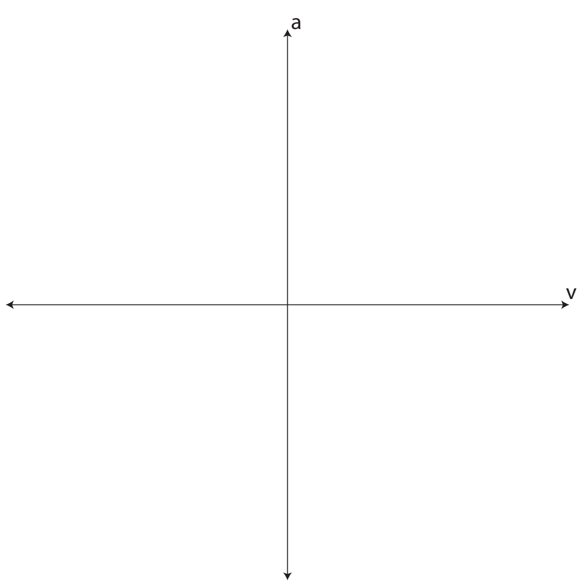
7. A 5-kg mass attached to a linear spring undergoes simple harmonic motion along a frictionless tabletop with an amplitude of 0.35 m and frequency of 0.67 Hz.

(a) Explain what the two criteria are for simple harmonic motion.

(b) Calculate the value of the spring constant.

(c) What is the ratio of the mass's acceleration when it is at half its amplitude to its acceleration when it is at full amplitude?

(d) Sketch a graph of the mass's acceleration as a function of its velocity for half of a cycle. Start when the mass is at its greatest positive amplitude. Mark the initial acceleration as a_0.



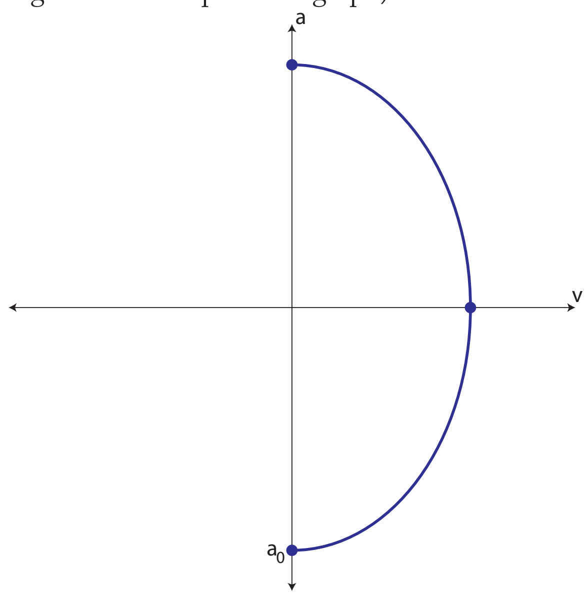
Answer:

(a) restoring force must be proportional to displacement; restoring force always opposes motion of mass

(b) $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \rightarrow k = (2\pi f)^2 m = 89\,{}^{N}\!/\!{}_{m}$

(c) The acceleration is proportional to the force which is proportional to the displacement according to Hooke's Law, therefore the ratio of a/a_0 is 0.5.

(d) When x is at a max, v is 0 and a is at a max. Acceleration is negative when displacement is positive. Velocity is positive for the entire half cycle. Velocity is proportional to the sine function and acceleration is proportional to the cosine function (out of phase), resulting in an elliptical plot (full credit for the three marked points regardless of shape of the graph).



EK: 3.B.3 Restoring forces can result in oscillatory motion. When a linear restoring force is exerted on an object displaced from an equilibrium position, the object will undergo a special type of motion called simple harmonic motion. 4.A.2 The acceleration is equal to the rate of change of velocity with time, and velocity is equal to the rate of change of position with time. 5.B.2 A system with internal structure can have internal energy, and changes in a system's internal structure can result in changes in internal energy.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 7.2 The student can connect concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

LO: 3.B.3.1 The student is able to predict which properties determine the motion of a simple harmonic oscillator and what the dependence of the motion is on those properties. 3.B.3.4 The student is able to construct a qualitative and/or a quantitative explanation of oscillatory behavior given evidence of a restoring force. 4.A.2.3 The student is able to create mathematical models and analyze graphical relationships for acceleration, velocity, and position of the center of mass of a system and use them to calculate properties of the motion of the center of mass of a system. 5.B.2.1 The student is able to calculate the expected behavior of a system using the object model (i.e., by ignoring changes in internal structure) to analyze a situation. Then, when the model fails, the student can justify the use of conservation of energy principles to calculate the change in internal energy due to changes in internal structure because the object is actually a system.

8. On a flat surface in Toledo, Ohio, a simple pendulum of length 0.58 m and mass 0.34 kg is pulled back an angle of 45° and released.

(a) Determine the theoretical period of the pendulum.

(b) When the period is measured (with a photogate, for 20 oscillations) it is found that the experimental period is 5 percent higher than expected. Repeated measurements consistently give similar values. What is the most likely explanation for this systematic error?

(c) What would be the period of this pendulum if it was in a free-fall environment? Explain.

Answer:

(a) $T_P = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{0.58m}{9.8 \, {}^{m}\!/_{s^2}}} = 1.53s$

(b) The most likely explanation is the theoretical equation is under-calculating the period. The equation used in part (a) is based on the small angle approximation and in the lab situation the angle is 45°, which is too large an angle to apply the small angle approximation. Consequently a large angle gives a slightly greater period than expected.

(c) There would be no period (or T = ∞) because the effects of gravity are not felt in free-fall.

EK: 3.B.3 Restoring forces can result in oscillatory motion. When a linear restoring force is exerted on an object displaced from an equilibrium position, the object will undergo a special type of motion called simple harmonic motion.

SP: 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 6.3 The student can articulate the reasons that scientific explanations and theories are refined or replaced. 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models.

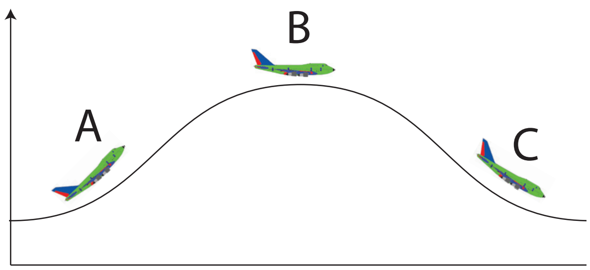
LO: 3.B.3.1 The student is able to predict which properties determine the motion of a simple harmonic oscillator and what the dependence of the motion is on those properties. 3.B.3.4 The student is able to construct a qualitative and/or a quantitative explanation of oscillatory behavior given evidence of a restoring force.

AP1 Oscillations

9. A group of students want to design an experiment where they test the period of simple pendulum as a function of the acceleration due to gravity, g. Part of their motivation is to verify the equation of the period of a simple pendulum: $T_P = 2\pi\sqrt{\frac{l}{g}}$.

They propose to bring their apparatus on NASA's Zero-g airplane that can create a range of values of g for short periods of time. Their proposal calls for three trials each for two situations where $g < 9.8\ m/s^2$ (where everything feels lighter than normal), $g = 9.8\ m/s^2$ and two situations where $g > 9.8\ m/s^2$ (where everything feels heavier than normal).

(a) If the Zero-g airplane moves in a continuous series of up-and-down parabolas, as shown below, identify at which position(s) in its trajectory someone would feel heavier than normal.



(b) If the simple pendulum the students bring consists of a 1.0 kg sphere with a string attached, discuss two other important factors that must be controlled throughout the experiment and how they will be controlled.

(c) Since the students have a limited number of trials for each value of g, they need a method of minimizing uncertainty in the measurement of the period of the pendulum. Discuss a method for doing so. Be sure to list necessary equipment and how it will be applied.

(d) Assuming that the students carry out the experiment and gather valid data, they want to graph their data. If they plot the variable T_p^2 on the y-axis, what variable should they plot on the x-axis so they get a straight line best fit?

(e) When they draw the line of best fit, the students calculate a slope of 9.8 m. What is the length of their pendulum?

Answer:

(a) Heavier than normal at positions A and C.

(b) Length of string can be controlled by securing one end to the sphere and the other one to a clamp system. The angle (amplitude) can be controlled by pulling back a predetermined (small) distance. Perhaps have a barrier to pull back to each time.

(c) Let the pendulum swing back and forth 5 to 10 times and time with a stopwatch or photogate system.

(d) Plot T^2 vs 1/g (accept T^2 vs. k/g where k is some constant value such as $4\pi^2L$ or L …)

(e) Assuming they plotted T^2 vs 1/g, the slope is proportional to T^2g. Using the pendulum equation $L = (T^2g)/4\pi^2 = \text{slope}/4\pi^2 = 0.25$ m

EK: 3.B.3 Restoring forces can result in oscillatory motion. When a linear restoring force is exerted on an object displaced from an equilibrium position, the object will undergo a special type of motion called simple harmonic motion.

SP: 4.2 The student can design a plan for collecting data to answer a particular scientific question. 5.1 The student can analyze data to identify patterns or relationships. 5.3 The student can evaluate the evidence provided by data sets in relation to a particular scientific question.

LO: 3.B.3.2 The student is able to design a plan and collect data in order to ascertain the characteristics of the motion of a system undergoing oscillatory motion caused by a restoring force. 3.B.3.3 The student can analyze data to identify qualitative or quantitative relationships between given values and variables (i.e., force, displacement, acceleration, velocity, period of motion, frequency, spring constant, string length, mass) associated with objects in oscillatory motion to use that data to determine the value of an unknown.